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has no minimum value in this domain. For so long as x is real and less than unity, $f(x)$ also is real and less than unity, and

$$f(x) < x,$$

that is, for every conceivable value of $f(x)$ of our domain we can construct another which is less.

IV.

Todhunter,[*] in spite of his extreme caution in dealing with demonstrations of a critical nature, also assumes the existence of a minimum without recognizing it. He himself outlines his argument in the words, "Since U^2+V^2 (that is, mod $f(z)$) is always a real positive quantity, if it can not be zero there must be some value which can not be diminished; but we shall now prove that if U^2+V^2 have any value different from zero we can diminish that value by a suitable change in the expression which is substituted for x, so that it follows that U^2+V^2 must be capable of the value zero, that is, U and V must vanish simultaneously." The argument as here stated really involves two unwarranted assumptions:

1. That the function U^2+V^2 necessarily possesses a minimum,
2. That a function which permits of being indefinitely diminished must necessarily approach the limit zero.

The second of these assumptions is removed in a postscript article, but the first is ignored.

It may be mentioned in conclusion that the assumption of the existence of a minimum occurs in the common source of the various proofs which I have cited, the so-called first proof of Cauchy,[†] or going back still further in Legendre[‡] who first developed the essential principles of Cauchy's proof.

THE UNIVERSITY OF NEBRASKA, *May, 1903.*

◆

A GENERAL NOTATION FOR VECTOR ANALYSIS.

By JOSEPH V. COLLINS, Stevens Point, Wis.

Vector analysis is now a little over half a century old. As compared with most other branches of mathematical thought its cultivation is very recent. As we look back over the history of the development of the mathematical notations we see that at first there was great diversity in the signs used, but that this diversity in time gave place to uniformity.

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[*] Todhunter, *Theory of Equations*, Chapter II.

[†] A. Cauchy, *Course d'Analyse*, Chapter XII, §1, Theorem I.

[‡] Legendre, *Theorie des Numbers*, 1re Part, §XIV; In the German translation by Maser, §119.

As matters now stand the several branches of vector analysis have each a separate notation. Thus there is the S and V notation of quaternions, the hooked bracket and sign of the complement of Grassmann's system, and Professor Gibbs's dot and cross, all denoting, if not always the same concepts, at least the same numerical quantities. Professor Gibbs's notation is distinctly an innovation and marks a tendency away from uniformity.

It is easy to infer why Professor Gibbs desired to throw overboard the quaternion notation. In the first place he desired juxtaposition to denote his so-called indeterminate products and then he wanted the places before and after his dyads unoccupied so that any new sign of operation desired could be inserted. This the quaternion S and V notation did not admit of. Again Professor Gibbs regards S and V as product rather than function symbols so that writing Sab and Vab is to him like writing ×ab for a×b. It is evident from this that the quaternion notation lacks symmetry. But there is still another objection to S and V as operation symbols. They are large and for that reason do not look well; nor can they be made quickly. Presumably it was for these reasons that Professor Gibbs gave up the quaternion notation entirely and introduced the dot and cross, the first to take the place of S and the latter of V.

Professor Gibbs has used a marked improvement in the general notation for vector analysis along another line in employing Clarendon type to denote vectors and the same letters in italics to denote their tensors. He thus avoids the use of Hamilton's cumbrous T and U. His system however is open to serious criticism. Besides being out of harmony with the quaternion symbols, this notation contains an important defect in that it uses up all of the signs available to signify multiplication, viz., juxtaposition, the dot, and the cross. Now there are other forms of product formation besides the dyadic and scalar and vector products, as for instance, the quaternion itself, or a product in which enters a trigonometrical function other than the sine and cosine. To have a system of notation to denote other forms of product, one would be under the necessity of inventing for them new arbitrary marks, which course would be open to serious objection. Fortunately all interests can be harmonized and ends met by the simple expedient of using small *letters* for the operation symbols and writing them *between* their factors.

The complete notation proposed may be described very briefly as the use of capitals to denote unit quantities, Clarendon type for vector quantities, the corresponding letters in italics for their tensors, and small letters, preferably initials, written a little above the line of writing, as symbols of operation, with the single exception of the indeterminate product, where no sign is used. Of course between two scalars or between a scalar and a vector no sign means multiplication. Thus ab=dyad or indeterminate product of vectors a and b.

$\mathrm{a}\,^{s}\mathrm{b}$=Sab=scalar product of vectors a and b.
$\mathrm{a}\,^{V}\mathrm{b}$=Vab=vector product of vectors a and b.
$\mathrm{a}\,^{t}\mathrm{b}$=scalar product of tensors of a and b times tangent of included angle.

$\mathrm{a}q\,\mathrm{b}$=quaternion product of vectors a and b.

$\mathrm{a}g\,\mathrm{b}$, or [ab]=Grassmann outer product of vectors a and b.

The writer has found this notation as convenient as the dot and cross. It is commended to the student of vector analysis as practical and as retaining the original Hamilton symbols. Evidently little effort is needed to pass from this to the old S, V, T, U notation. Finally, it is adapted to the needs of the general science of vector analysis and admits freely of the introduction of new products.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

167. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

A traveler notices that $m=2\frac{1}{2}$ times the number of spaces between the telegraph poles that he passes in a minute is the rate of the train in miles per hour. How far apart are the poles?

Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let n=number of spaces passed per minute, d=distance between poles.

Then mn miles per hour$=\dfrac{mn\times5280}{60}=88mn$ feet per minute.

$\therefore\ dn=88mn$, or $d=88m$ feet. When $m=2\frac{1}{2}$, $d=220$ feet.

168. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

If I buy for $m=10\%$ and $n=5\%$ *less* I shall gain $p=15\%$ and $q=5\%$ *more*. What is my rate of gain?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

This problem is not clear. I interpret it to mean: If I buy for $100\times90\times95=85.5\%$ instead of for 100%, and sell for the same price as I sold when I paid 100%, I would gain $100\times115\times105=120\frac{3}{4}\%$, $-100\%=20\frac{3}{4}\%$ more than I did gain. $85\frac{1}{2}\%=\frac{171}{200}$, $20\frac{3}{4}\%\frac{83}{400}$, $\frac{83}{400}+1=\frac{483}{400}$. $(\frac{483}{400}\times171-200)\div(200-171=(\frac{82593}{400}-200)\div(200-171)=6\frac{193}{400}\div29=\frac{2593}{11600}$, $\frac{2593}{11600}$ of $100\%=22\frac{41}{116}\%$.